1. The relation of predication

Aristotle in *Categories* 5 introduces a sort of relationship between categorical formulas:

It is clear… that if something is said of a subject both its name and its definition are necessarily predicated of the subject. For example, man is said of a subject, the individual man, and the noun is of course predicated (since you will be predicking man of the individual man), and the definition of man will be predicated of the individual man (since the individual man is also a man). Thus, both the noun and the definition will be predicated of the subject.1

In the text a common noun occurs in the subject position as name of a second substance and as predicate in a different categorical statement. The link between the two formulas can be formalized by the following biconditional:

B1: $X \varphi 'a$ iff a is (an) x

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1 *Categories* 2a19 (trans. Ackrill); Boethius’ transl.: *In Cat.* 184 C: “Alia autem omnia aut de subjectis dicuntur principalibus substantiis, aut in subjectis eisdem sunt… animal de homine praedicatur ergo de aliquo homine, nam si de nullo … neque omnino de homine.”
Where $\varphi$' is a predicate constant standing for a relation of predication. One instance of the principle B1 is the following:

Man is said of Plato if and only if Plato is a man

The relationship between second substances can be defined in a similar way:

\[ B2: X \varphi \equiv Y \text{ iff } *\text{every } y \text{ is (an) } x \]

We can trace B2 in Aristotelian texts where the analytical formulas like ‘A is said of each B’ are put in correspondence with ordinary categorical statements. Schematic letters ‘X’, ‘Y’ occurring in B1-2 can be interpreted in various way. Logicians in medieval and post-medieval era evaluate a wide range of possibilities: sets, properties, type words, essences… Every interpretation has consequences on the logical and metaphysical ground and gives rise to specific problems. One may prefer one or the other interpretation according to his own philosophical opinions. Some of these choices presuppose the adoption of a certain ontology and metaphysical position on the problem of universals (understood in the Porphyry sense).

Some modern interpreter\(^2\) has noted that Aristotle fails to recognize different predication relations. To correct this “elementary conceptual mistake”

\[^2\text{Ackrill 1963; Lewis 1991, in particular pp. 73–78; De Rijk 2002, in particular par. 4.25, pp. 380–386; Guha 2010.}\]
that “the father of formal logic should not make”\textsuperscript{3} different stratagems have been employed. But firstly, how many relations do we actually have to consider? It seems to me that there are potentially four relations at play: predication in general between specie and genera, predication between individual things and species/genera, saying-of between species and genera, saying-of between individual things and species/genera. In defining the predication relation, we have considered two different formulas for two potentially different relations. For those who do not like the idea that Aristotle considered two different relations of said-of, at least three possibilities arise: the first one is simply choosing between $\varphi' e \varphi''$; the second defining a relation in terms of another one;\textsuperscript{4} the third pointing at particular interpretations. I shall say something about the third way. It seems to me that what De Rijk writes on this subject, although partial, is indicative of the problem. The Dutch scholar gives a metalinguistic reading of the passage in which Aristotle speaks about the names of things that are said-of.\textsuperscript{5}

Given that the domain of the predication relation be a set of names, not much is needed to bypass the transitivity problem. According to De Rijk we

\begin{itemize}
\item Guha proposes some solutions to restore univocity, avoiding at the same time paradoxical consequences. Among others: a) the predication relation $\varphi$ can be defined in a disjunctive way $(x \varphi' y \lor x \varphi'' y)$, b) one relation can be defined in terms of the other, c) $\varphi$ can be interpreted as a kind of iterated exemplification: Socrates is an example of man, man is an example of animal and Socrates is an example of an example of animal.
\item In a parasitic way, regarding the definition of class inclusion, one could propose something like: $X \varphi'' Y$ iff $\forall a Y \varphi' a \rightarrow X \varphi' a$.
\item Cf. De Rijk 2002: “while Ackrill correctly states that the chapter deals with the transitivity of the ‘said of’ relation with regard to genera and species, his failure to see that the focus is on naming, not sentence predication, leads him to the mistaken view that Aristotle does not distinguish between the transitivity relation of an individual to its species or genera and that of a species to its genera” (p. 381). The scholar at the same time recognizes that the transitivity principle must be implemented by conditions on the choice of predicates (par. 5.51, pp. 518–521), conditions that should be incorporated on the right side of B formulae.
\end{itemize}
should assume the B variants:

B1(dR): ‘X’ \( \varphi \cdot 'a' \) iff \( a \) is a x iff (‘a’ denotes and) what ‘a’ denotes is denoted also by ‘X’

B2(dR): ‘X’ \( \varphi \cdot 'Y' \) iff every y is a x iff (‘Y’ denotes and) what ‘Y’ denotes is denoted also by ‘X’

Where ‘X’, ‘Y’ are common nouns and ‘a’ is a proper name. For instance ‘man’ \( \varphi \cdot 'Socrates' \), holds, because what ‘Socrates’ names (the individual Socrates) is also named by ‘man’; ‘animal’ \( \varphi \cdot 'man' \) is true as well because all what ‘man’ names is also named by ‘animal’; it does not happen that ‘species’ \( \varphi \cdot 'man' \), because what ‘man’ names is not named by ‘species’. Is it all right about principle B? Yes, but at the cost of: a) underestimating the distinction between the naming (denoting) of a proper name and common noun; b) carrying out an analysis which mixes syntactic and semantic levels; c) sharing a nominal interpretation of species and genera, interpretation that has an important history together with strong opponents highlighting its faults; d) accepting a later theory of truth, i.e. the medieval identity theory.

2. Different readings

The biconditional stated above can be read as double material conditional, as nominal definitions (let’s call it stipulative reading) or as explanations (providing some kind of reason why). An ordinary categorical sentence does
not correspond directly to a sentence of a theory of predication, but there exist conditions under which some categorical sentences correspond to formulas which are supposed to mirror the metaphysical structure of reality. So, there are different ways to read the equivalence B: i) material: only the agreement in true value is considered; ii) stipulative: looking at the introduction of a new concept; iii) considering a reason why a sentence can be true. Examples of (i): \(x+y=y+x\) iff \(x\cdot 0=0\); of (ii) \(A \subseteq B\) iff \(\forall x \in A \rightarrow x \in B\); of (iii): two bodies attract each other in space (observable fact) if the law of physics provides for it.

In the explicative readings establishing the logical form for the equivalences B is a problem, and, in general, rendering the relationship between explicandum and explanans. The Aristotelian text quoted before: “for example, man is said of a subject, the individual man, and the name is of course predicated (since you will be predicing man of the individual man...)” leads to think that only one side of the bi-conditional holds, but things are more complicated. If we employ an equivalence expression we probably need to enrich the premise, since the truth of a general law does not guarantee that an instance actually occurs; viceversa, the phenomenon occurrence ‘entails’ the law, but one might wonder if this form of entailment be formalizable with a material implication sign (in the text quoted we find ‘since’).

The stipulative reading in the ancient and medieval period is rather undercurrent. A predication theory would be needed to have a complete landscape, but this does not exist. We only find a (fragments of) the theory of definitions and that of whole and parts. In the former case Medieval scholars employ Latin common nouns in a position that seems to require a translation by a(n elliptical) definite article, for example a possible translation for the Latin ‘homo est animal rationale mortale’ could be: ‘(the) man is the animal rational
mortal’. In the latter case Latin scholars employ plural forms or phrases like ‘collectio (collectionum) + genitive’.

B formulae, read in a stipulative way, constitute a kind of bridge toward (proto) theories of sets, generic entities and so on. In order to walk across the bridge, blurry intuitions about natural language sentences need to be considered. Difficulties soon arise in this ‘middle land’; they essentially depend on the acceptance of something like Frege’s abstraction principle. The issue of paradoxes is not on the horizon, but in the context of the comments to Porphyry the one of anomalies that depend on the logical type of predicates is relevant. Let me explain the point with a fragment of the graph of the predication relation, where the universals are seen as sets (black arrows= $\wp'$, red arrows= $\wp''$):
The only composition allowed is that between red arrows \( \varnothing'' \) (it does not occur), or between \( \varnothing'' \) and the black arrows \( \varnothing' \) (in this order). It is worth noting that, according to B principles, the set of souls, for instance, is a set; it follows that the set of souls has a relation \( \varnothing' \) with the set of all sets; moreover, the set of souls is an abstract entity, then it has a relation \( \varnothing' \) with the set of incorporeal things. On the other hand, the same set is contained in the set of incorporeal things, i.e. has a relation \( \varnothing'' \) with it. As regards other interpretations of universals the same thing applies, as to, for example, generic entities or properties (forms). The collection of collections (this is a medieval notion!) is an incorporeal entity, but the collection of incorporeal entities is an element of the collection of collections, and so on. Other interpretations of universals (as, for example, generic entities or properties) give rise to similar difficulties.

3. Porphyrian questions

In the Isagoge Porphyry aims to explain the technical notion of genus, species, difference, proprium and accident. These terms have a different logical status. ‘Species’ and ‘genus’ refer to the relation between a singular or universal thing and other universal things; ‘proprium’ means a property of accidental properties owned by things in an exclusive way; ‘accident’ expresses a property of properties that something can have or have not. The difference has a more complicated logical status: a property is said difference if together with a universal it gives – in same sense of giving, possibly an operational one – another universal. Let me quote the famous passage in which Porphyry throws the stone and hides his hand, so to speak.
I shall omit to speak about genera and species, as to whether they are separate from, or in, perceptible things, and subsist about these, for such a treatise is most profound and requires another more extensive investigation.6

This is the most tangled question Medievals investigate. It is not about, at least at first glance, properties, or types of objects, but about genera and species, that is to say non-singular typed objects ordered according to a certain relation of predication. In what sense are B equivalences an acceptable answer to the Porphyry’s queries? two questions arise:

(I1) what is the set of entities on which the relation of predication can be defined.

(I2) what is the meaning of ordinary statements (like ‘every man is animal’)?

These issues have to do, or have prevalently to do with an explicative reading of type B equivalences. From I1’s point of view, the existence (being) of genera and species as adjunctive entities in the domain of individual things is a problem, so it is for the cardinality of the enlarged interpretation domain; the truth/answerability of ordinary categorical sentences is not a problem yet. A way to explain the sense of ϕ relation is to establish the existence and type of objects on which it is defined. There is a subtle difference between this task and that of who endorses a stipulative reading. The former consists in proving, or arguing for, the existence of something, the second assumes the existence of elements belonging to the interpretation domain.

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6 Isagoge, 610. Boethius’ translation is the following: “Mox de generibus et speciebus illud quidem siue subsistunt siue in solis nudis purisque intellectibus posita sunt siue subsistentia corporalia sunt an incorporalia, et utrum separata an in sensilibus et circa ea constantia, dicere recusabo. Altissimum enim est huiusmodi negotium et maioris egens inquisitionis,” In Isagog., 159, 3–9.
The problematic object for I2 is the ordinary predication: when I assert a categorical sentence such as ‘Socrates is wise’ or ‘every man is an animal’, what justifies my assertion? The question is not (only) about the existence of the wise man, wisdom, or the human being, but on how these things interact to build a fact. In Porphyry’s perspective: if genera and species exist and are abstract entities, how does the abstract relate to the concrete? Is the abstract universal separate or subsistent in the perceptible thing? Typically, one focuses on the difficulties depending on the presence of the universal in the individual entities of which the noun is predicated. Not all interpretations of the predication appear to be right in justifying ordinary predication.

Looking at the B equivalences from the perspective of I2, one implicitly is led to assume a metaphysical point of view on the predication, and seeks an interpretation of the predication in terms of a real relation between things that are supposed to be real. Such an interpretation is supplied by realist theories which deal with the inherence of (the meaning of) the predicate to (the meaning of) the subject. This does not seem to work well for all the interpretations of categorical terms: the proto-set theory developed by the Medievals is barely usable for metaphysical purposes; it does not make much sense to say that the basis of the truth of ‘Socrates is wise’ is Socrates belonging to the set of wise things. Abelard, for instance, criticizes the mentioned theory as it does not allow a semantic interpretation of terms and does not meet Porphyry’s setting, but not because it involves an inconsistent view of the world. Despite this, there are authors, also Medieval, which consider a set (proto-) theoretic reading as a way to orient themselves on the problem of universals.

Another question about I2, has to do with the problem of what the ex-
planation is about. We said that natural language sentences must be evaluated, but the problem is if the superficial form of a categorical sentence is a faithful witness of its meaning. Early on in the Middle Ages someone considered the task of unearthing a logical form of sentences corresponding, in some sense, to the structure of reality. This move is apparently independent of the one interpreting predication as a relationship between singulars and universal entities, because the ‘superficial’ form of a sentence seems to involve that. One can also think that sentences formulated in a rigorous way are the ones having clearly identifiable truth conditions. In this perspective, sets do not play the role of surrogates of ordinary predicates, but of objects about which we can say something true or false.

A kind of mantra, which has its priests also in the Middle Ages, recites: natural language is full of vagueness and ambiguities, what we want to say, when we want to say something of meaningful, is not correctly expressed by the superficial form of the sentences but by their logical form. The problem is that of maturity. To resolve that problem one can try to build a mature and reliable language looking at what actually exists and how existing things contribute to set up facts/events of a certain sort. To carry out this task one must prefer a way of knowing or a language as opposed to other ways or languages. Let me to give an example: let us assume that only concrete particulars and sets constructed, in some sense, from concrete particulars, exist, so a ‘mature’ language should contain designators for these objects. Comparing a mature language with an immature one, from an ‘external’ perspective, in the words of Carnap, presupposes a negative valuation of the latter and such depreciation leads to say its statements are not justifiable. Actually, I think something more can be said, at least conjecturally: every failed justification of
the natural language and its supposed truths passes through a (failed) tentative of evaluation.

Some conclusions: it seems of a certain interest to consider the difficulties that emerge from certain interpretations of $B$ equivalences. One possible suggestion is to seek a way to avoid set paradoxes by considering antisymmetric $\varnothing$ relations (one thing is not related to another and vice versa) and exclusive (if one thing is related $\varnothing'$ to another), cannot be in relation $\varnothing''$). As to the possible readings of Aristotelian formulas, what we have said so far obviously does not exhaust the subject. It is possible to trace stipulative and explicative principles that are not utilized in the justification of ordinary predication. Some philosophers interested in the solution of Porphyry’s problem are also involved in other – perhaps non-canonical – more abstract questions.

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In Cat. = BOETHIUS, In Categories Aristotelis, in PL 64.

In Isagog. = BOETHIUS, In Isagogen Porphyrii Commenta, ed. S. Brandt, Wien-Leipzig, Temsky – Freytag 1906
